X is the coordinate normal to the wave front; ρ_0 is the density of the medium; ρ is the density of the particle; C_0 is the speed of sound in the medium; C is the speed of sound in the particle; $\dot{X}$ is the particle velocity; $\ddot{X}$ is the particle acceleration; t is the time; and η is the liquid viscosity.

REFERENCES

1. L. Bergmann, Ultrasonics [Russian translation], IL, 1957.

2. L. P. Gor'kov, DAN SSSR, 140, no. 1, 88, 1961.

3. L. K. Zarembo and V. A. Krasil'nikov, Introduction to Nonlinear Acoustics [in Russian], Izd. Nauka, Moscow, 1966.

4. V. G. Levich, Physicochemical Hydrodynamics [in Russian], Fizmatgiz, Moscow, 1959.

5. H. B. Dwight, Tables of Integrals and Other Mathematical Data [Russian translation], Izd. Nauka, Moscow, 1966.

19 May 1967

Institute of Chemical Engineering, Moscow

RAYLEIGH EQUATION FOR THE GROWTH OF A GAS BUBBLE UNDER CONDITIONS OF FINITE LIQUID VOLUME

E. D. Kurtsman

Inzhenerno-Fizicheskii Zhurnal, Vol. 14, No. 1, pp. 165–167, 1968

UDC 532.51

An equation is presented for the growth of a stationary spherical gas bubble in a finite spherical liquid volume, for which the familiar Rayleigh equation is the zero-order approximation. The results of computer solutions of the derived equation and the Rayleigh equation are compared.

In [1] Rayleigh derived an equation for the growth of a stationary spherical gas bubble in an infinite volume of liquid. Thus, from the equations of hydrodynamics and continuity in a spherical coordinate system

$$\rho\left(\frac{\partial v}{\partial t}+v\,\frac{\partial v}{\partial r}\right)=-\frac{\partial P}{\partial r},$$

$$\frac{\partial v}{\partial r}+\frac{2v}{r}=0$$

with boundary condition $v|_{r=R}=\dot{R}(t)$, where, as usual, $\dot{x}\equiv dx/dt$, one obtains

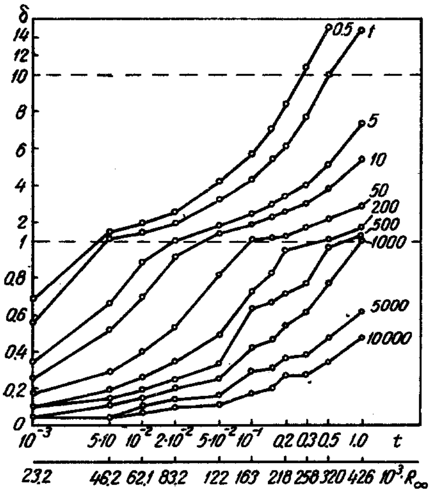
$$\frac{\ddot{R}R^2+2R\dot{R}^2}{r^2}-\frac{2R^4\dot{R}^2}{r^5}=-\frac{1}{\rho}\,\frac{\partial P}{\partial r}. \qquad (1)$$

Then Rayleigh, integrating (1) from $r = R$ to $r = \infty$, obtained his familiar equation

$$R\ddot{R}+\frac{3}{2}\dot{R}^2=\frac{1}{\rho}\,(P_R-P_\infty). \qquad (2)$$

Equation (2) has been used by many authors (for example, [2–5]) in connection with problems involving the growth of a bubble in a liquid. However, the finiteness of the liquid volume has not been taken into account.

To estimate the effect of this factor on the bubble growth and to compare with the Rayleigh solution, we



Deviation of the solutions for R_Q in Eqs. (4) with Q = var from the solution R_∞ of the Rayleigh equation (for $Q = \infty$). Plotted along the axis of abscissas, to a logarithmic scale, are the time t (sec) and the corresponding radius R_∞ ($\cdot 10^3$, cm); along the ordinate axis, to a variable scale, we have the deviation in % of R_∞ from R_Q for the corresponding t. $\delta = (R_Q - R_\infty)\cdot 100/R_Q$.

formulate the simple following problem: at time $t = 0$ a spherical gas bubble of radius R_0 is formed and begins to grow at the center of a spherical volume Q of liquid with infinite permeability at the boundary, the growth of the bubble being accompanied by an increase in the external radius of the liquid sphere. For simplicity, the pressure at the external boundary of the liquid is assumed constant and equal to zero: $P_\infty = 0$.

Then, integrating (1) from $r = R$ to $r = (3Q/4\pi + R^3)^{1/3}$, we have

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 - \frac{\ddot{R}R^2 + 2R\dot{R}^2}{\left(\frac{3Q}{4\pi} + R^3\right)^{1/3}} + \frac{R^4\dot{R}^2}{\left(\frac{3Q}{4\pi} + R^3\right)^{4/3}} = \frac{P_R}{\rho}. \quad (3)$$

Thus, the Rayleigh equation is the zero-order approximation ($Q = \infty$) of Eq. (3).

Retaining terms with Q to the power $-1/3$ in the Taylor expansions of the denominators in Eq. (3), we obtain the equation for the first approximation

$$R\ddot{R} - \left(\frac{3Q}{4\pi}\right)^{-1/3} R^2\ddot{R} - 2\left(\frac{3Q}{4\pi}\right)^{-1/3} R\dot{R}^2 + \frac{3}{2}\dot{R}^2 = \frac{P_R}{\rho}. \quad (4)$$

This equation was solved on a M-20 computer for a simple law of variation in P_R

$$P_R = P_0\left(\frac{R_0}{R}\right)^3.$$

We assumed $\rho = 1$ g/cm^3, $R_0 = 10^{-3}$ cm, and $P_0 = 2 \cdot 10^4$ N/m^2. A solution was obtained for $Q = \infty$ (Rayleigh equation (2)) and for Q from 10^4 to 0.5 cm^3 (for Eq. (4)).*

*A calculation was also performed with the given values of ρ, R_0, P_0 and $P_\infty = 0$ for the second-order approximation, retaining terms with Q to the power $-4/3$ in the Taylor expansion of the denominators. The results were in complete agreement with the calculations for the first-order approximation, which confirms the possibility of stopping at the first approximation in this particular example.

The results of the calculations are presented in the figure; from these one can estimate the error given by the Rayleigh solution as compared with the solution of Eq. (4), which is more accurate for a finite volume of liquid.

The form of the curves should remain qualitatively the same for a slowly rising spherical bubble in a heated liquid (for example, in the initial stage of bubble growth during boiling), since the hydrodynamic terms in the equation of dynamic equilibrium for such a bubble play a part which is small as compared with the terms responsible for the surface tension and rate of evaporation of liquid inside the bubble [5]. Hence criteria can be derived for the possibility of neglecting the effect of the walls and other bubbles on the growth of a given bubble in boiling studies and also to determine the minimum vessel dimensions necessary for the experimental investigation of bubble growth in a liquid.

NOTATION

R is the radius of a gas bubble; P_R and P_∞ are the pressure at the inner and outer boundaries of the liquid, respectively; Q is the volume of liquid, cm^3; P_0 is the initial pressure in the gas bubble, N/m^2; $v(r, t)$ is the velocity field in liquid; ρ is the density of the liquid.

REFERENCES

1. Rayleigh, Phil. Mag., **34**, 94, 1917.
2. H. Lamb, Hydrodynamics [Russian translation], GITTL, Moscow-Leningrad, 1947.
3. M. S. Plesset and S. A. Zwick, J. Appl. Phys., 25(4), 493, 1954.
4. H. K. Forster and N. Zuber, J. Appl. Phys., 25(4), 474, 1954.
5. Yu. A. Aleksandrov et al., Bubble Chambers, ed. N. B. Delone [in Russian], Gosatomizdat, Moscow, 1963.

18 May 1967

Institute of Theoretical and Experimental Physics, Moscow.